



Tracer transport in the Unified Model

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The plan of attack

- The Unified Model
- Some notation and nomenclature
- The semi-Lagrangian scheme
- ENDGame
- Does it matter?
- SLICE – recovering conservation
- Conservation in LAMs
- GungHo!
- Bibliography
- Transport options in ROSE



THE UNIFIED MODEL



Unified Model

Brown et al. (2013)

■ Operational forecasts

- Mesoscale (resolution approx. 1.5km)
- Global scale (resolution approx. 10km)

■ Global and regional climate predictions

- Resolution around 120km
- Run for 10-100-... years

■ Seasonal predictions

- Resolution approx. 60km

■ Research mode

- Resolution 1km - 10m

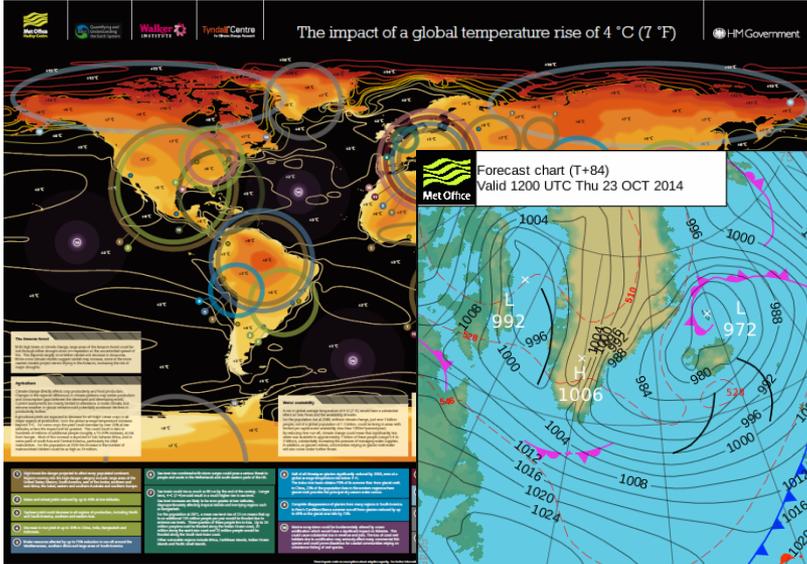
> 25 years old



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The consequence of unification

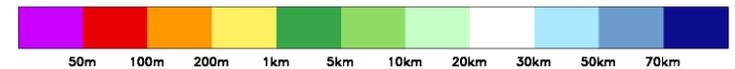
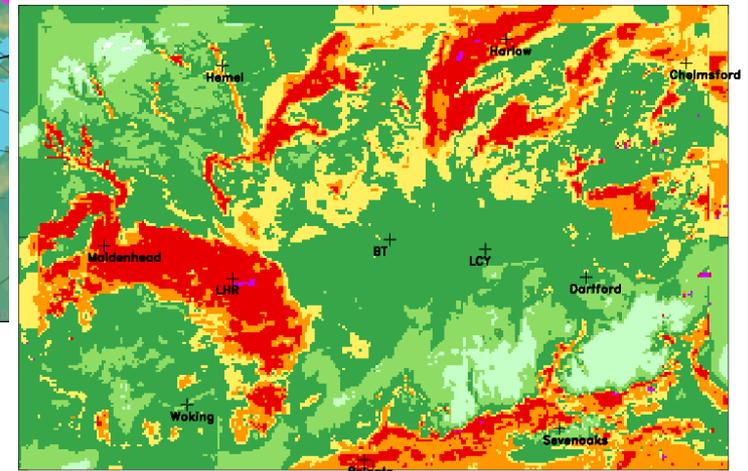
300 km



30 km

A factor of 1000 between these

300 m

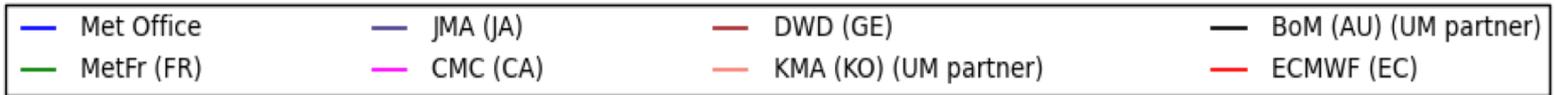


...the same scheme has to continue to work

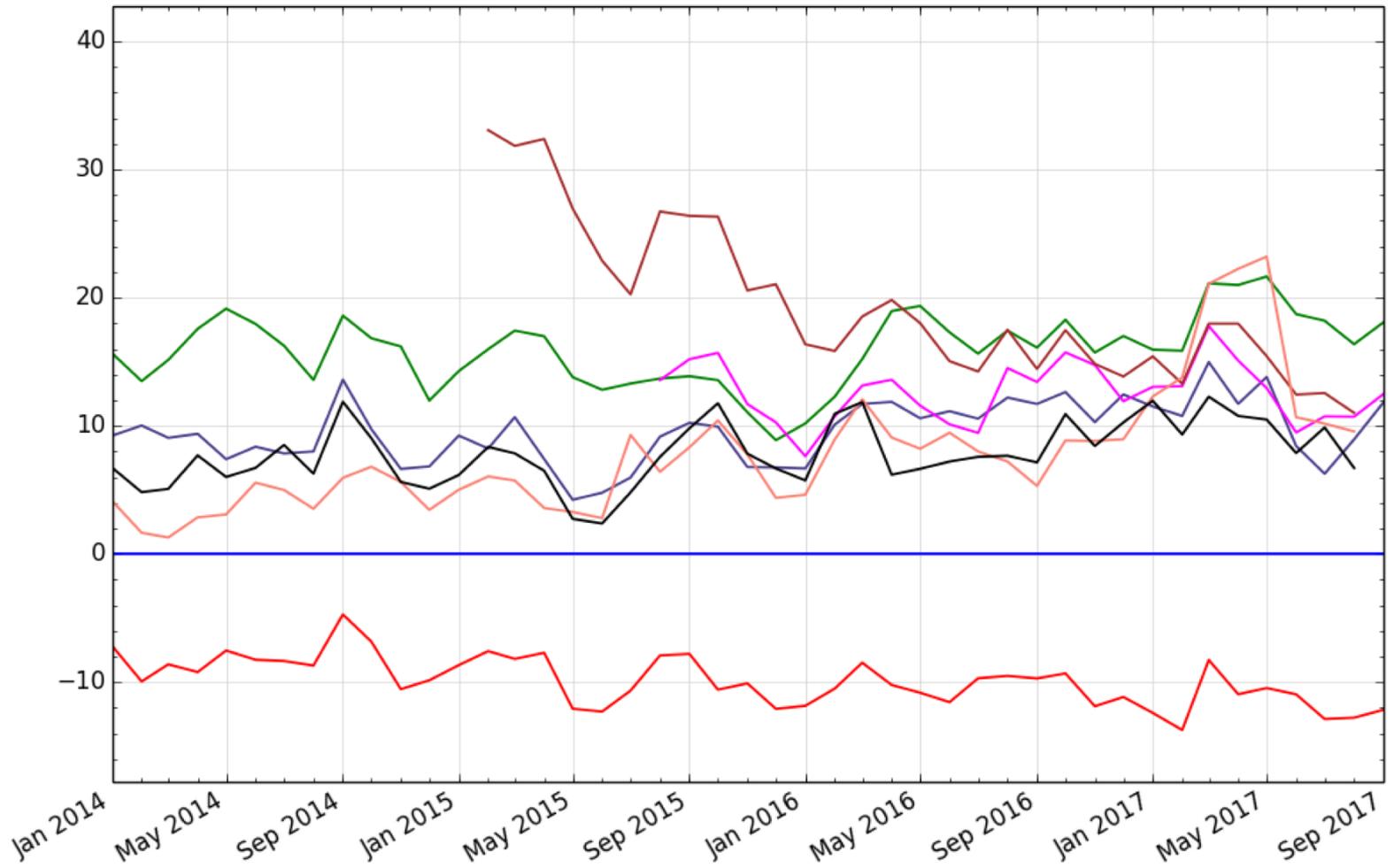


Global model cf. other centres

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Skill* Difference (%) relative to Met Office



* Parameters: PMSL, 500hPa GPH, 250hPa/850hPa Winds; Range: T+24 to T+120



NOTATION AND NOMENCLATURE



Notation

- Let ρ_X denote the ***density, concentration, or mass per unit volume*** of species X
- Let ρ_d denote the density of ***dry air***
- Then $m_X = \rho_X/\rho_d$ is the ***mixing ratio*** of species X
- By definition $m_d = 1$



Conservative form

Densities/concentrations transported according to:

$$\frac{\partial \rho_X}{\partial t} + \nabla \cdot (\mathbf{U} \rho_X) = 0 \quad \text{Eulerian flux form}$$

$$\frac{D}{Dt} \left(\int_V \rho_X dV \right) = 0 \quad \text{Lagrangian form (V=air parcel)}$$



Advective form

Mixing ratios/parcel labels (e.g. age of air, mass of air parcel) are transported according to:

$$\frac{\partial m_X}{\partial t} + \mathbf{U} \cdot \nabla m_X = 0 \quad \text{Eulerian form}$$

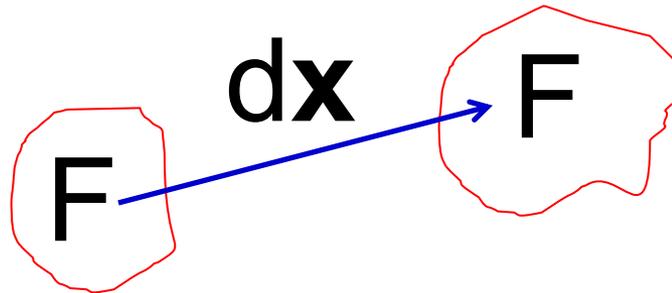
$$\frac{Dm_X}{Dt} = 0 \quad \text{Lagrangian form}$$



THE SEMI-LAGRANGIAN SCHEME

From nature to a computer

- $DF/Dt=0$ a natural form
- Integrate along the path a fluid parcel follows



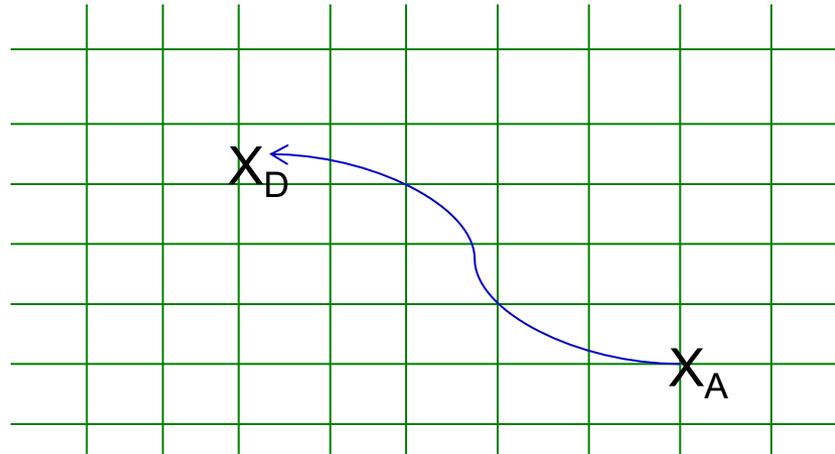
- $F(\mathbf{x}+d\mathbf{x},t+dt) = F(\mathbf{x},t)$ where $d\mathbf{x}/dt=\mathbf{U}$



Lagrangian & semi-Lagrangian

- **Lagrangian** model simply tracks air parcels
- This is the basis of the NAME model for plumes etc
- But, generally end up with very inhomogeneous distribution, requires interpolation/aggregation to where need answer
- **Semi-Lagrangian** schemes try to maintain the benefits of Lagrangian approach but on Eulerian grid

Semi-Lagrangian



- Arrival point, X_A , always a grid point
- Departure point, X_D , in general anywhere
- Two steps:
 - Evaluate trajectory, i.e. where X_D is relative to X_A
 - Evaluate transported field at X_D

Staniforth and Côté (1991)



Benefits

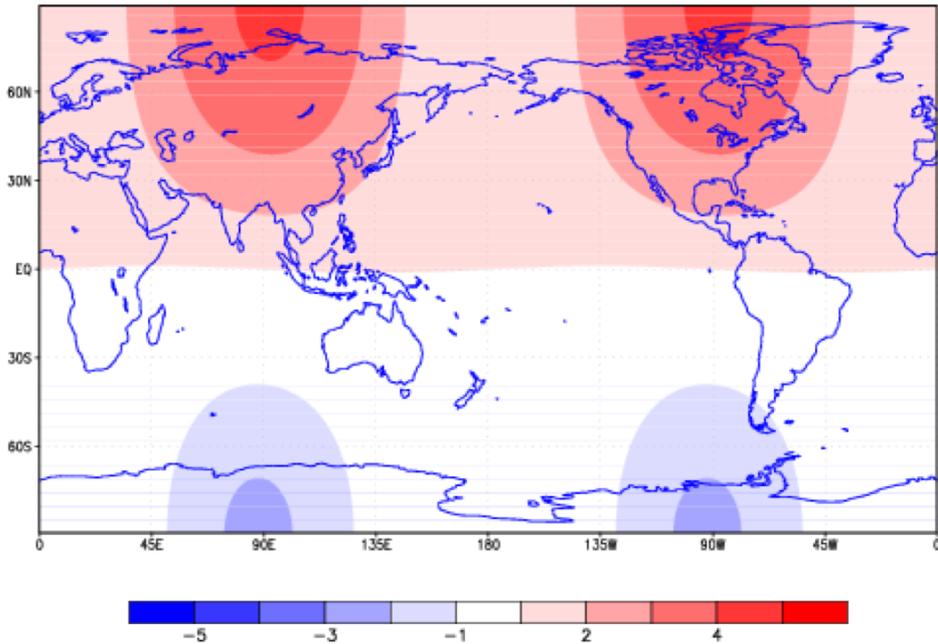
- Excellent **dispersion**
 - Captures well the speed of propagation of waves
 - Key for good weather prediction
- Appropriate level of **scale selective damping**
- Excellent **stability**
 - Depends on physical (inverse) time scale $d\mathbf{U}/d\mathbf{X}$, not numerical (inverse) time scale $\mathbf{U}/\Delta\mathbf{X}$
 - Particularly beneficial in large scale flows (cf. jets)
 - And in polar regions (operationally, polar $\Delta\mathbf{X}=12$ m, $dt = 4$ mins, and CFL = 1 for $U=5$ **cm/s!**)



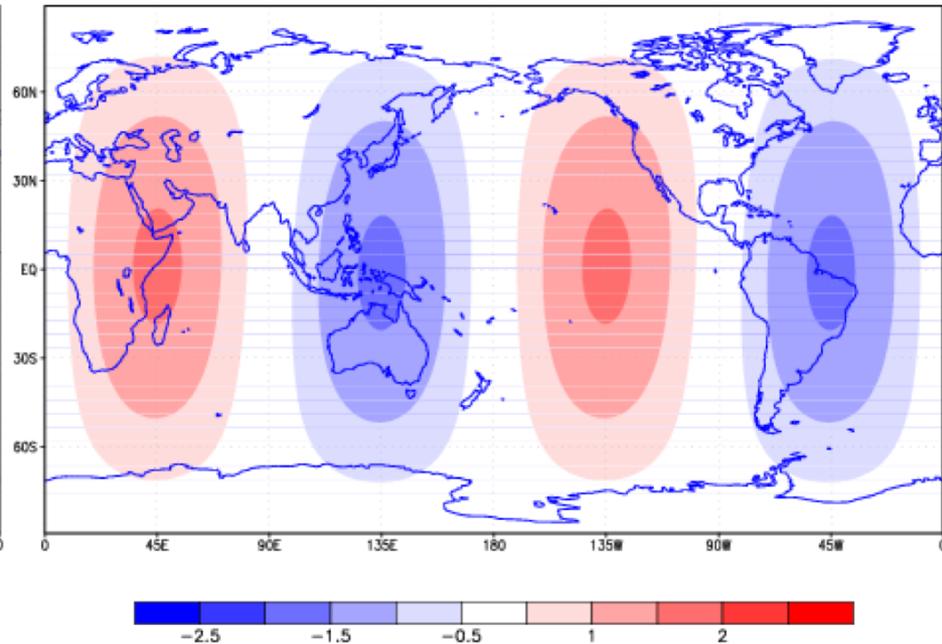
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An example

CFLX



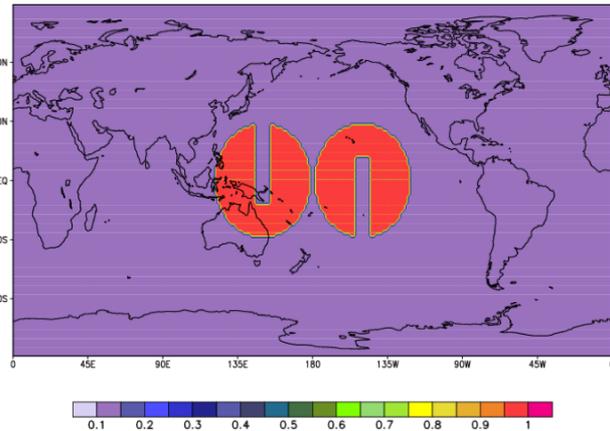
CFLY



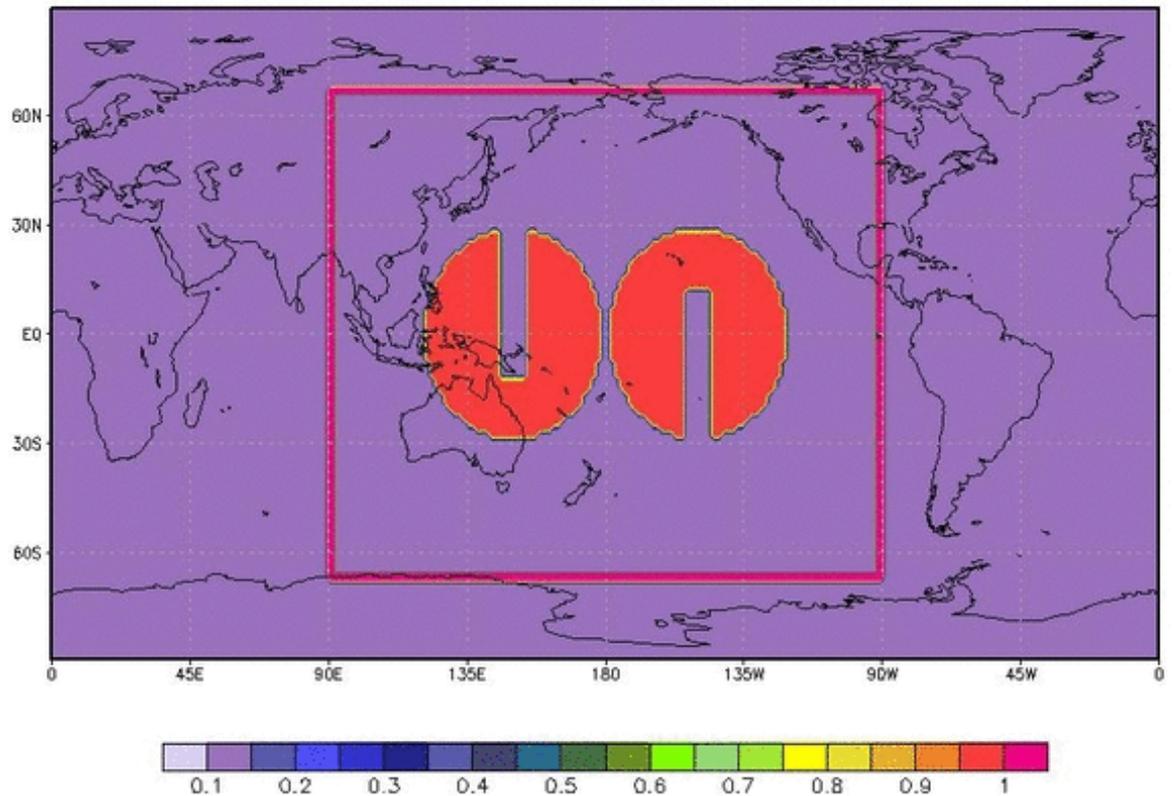
Kohei Aranami (MetO/JMA)

Slotted cylinder test case

Initial Conditions



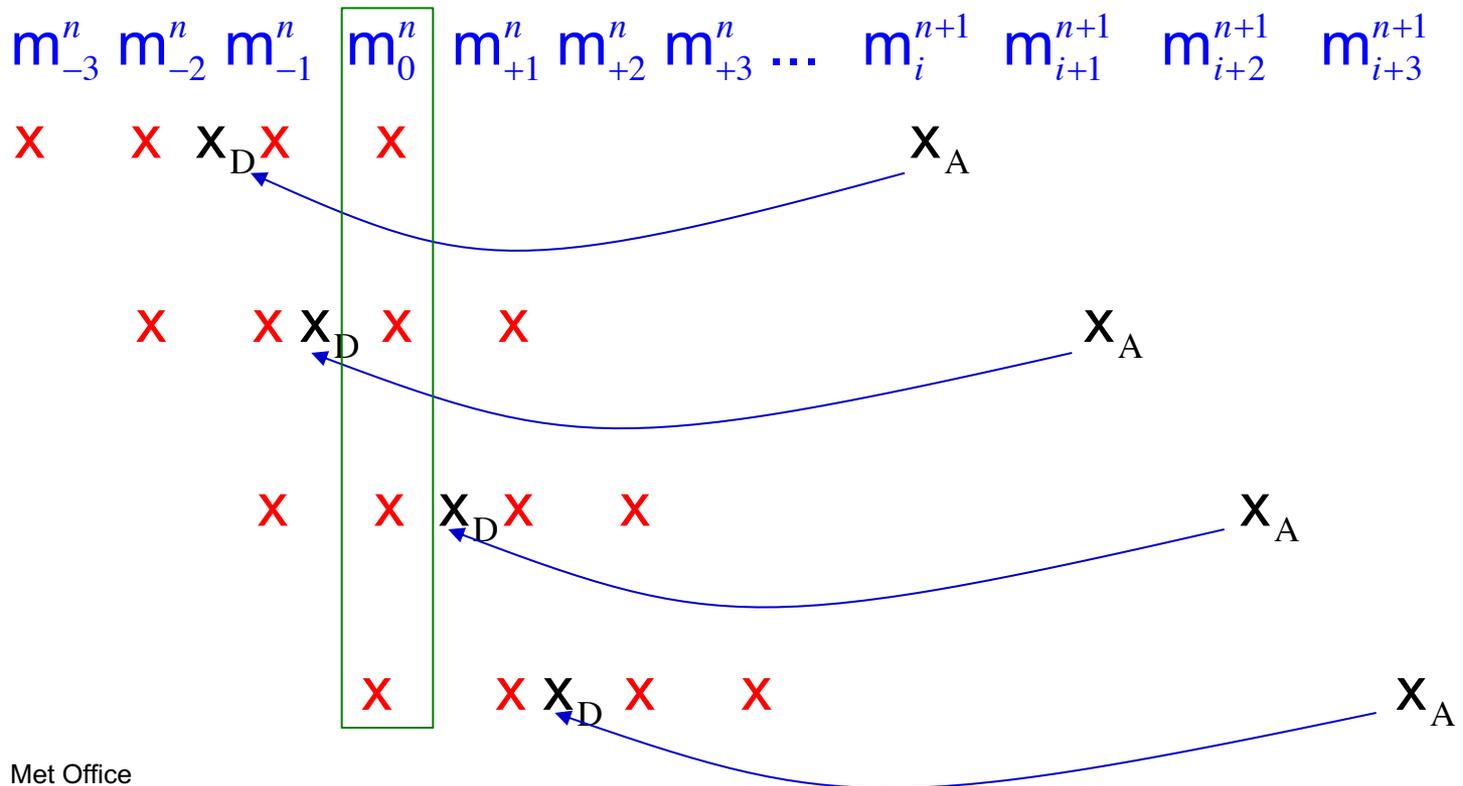
SL-QMSL PLF : Fields at t=001/241



Kohei Aranami (MetO/JMA)

Disbenefits

- Lack of locality due to large time step, means departure point can be long way from arrival
- Conservation - consider cubic interpolation:





Conservation

- Even in case of interpolating mass (so don't have to worry about density variations and non-uniform grid spacing), require:

$$\sum_i m_i^n = \sum_i m_i^{n+1} = \sum_i \left(a_j m_{j(i)-2}^n + b_j m_{j(i)-1}^n + c_j m_{j(i)}^n + d_j m_{j(i)+1}^n \right)$$

- For this to hold independent of mass distribution

$$\left(a_{i+2} + b_{i+1} + c_i + d_{i-1} \right) m_i^n = m_i^n$$

which is only true if wind is uniform

- [Cf. $a_i + b_i + c_i + d_i = 1$]



ENDGame: Even Newer Dynamics for General atmospheric modelling of the environment

(Operational since 2014; Wood et al 2014)



Transport in ENDGame I

- Semi-Lagrangian scheme applied to all variables
- Special handling of vector aspects for wind
- Lagrangian interpolation:
 - Horizontal
 - Bi-cubic for all variables
 - Vertical
 - Cubic for wind components
 - Cubic-Hermite for potential temperature and moisture variables
 - Quintic for all other tracers



Transport in ENDGame II

- Conservation:
 - Priestley algorithm (optionally) applied to moisture and tracer variables **and** potential temperature
- Monotonicity:
 - Bermejo and Staniforth (optionally) applied to moisture and tracer variables **and** potential temperature



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Dry mass conservation

- Without mass fixer relative change in total mass per time step is $O(10^{-5})$
- \Rightarrow apply multiplicative fixer every time step
- Important that it preserves potential energy

- Achieved by:

$$\rho^{n+1} = (A + Bz) \rho^*$$

- A and B chosen such that

$$\sum \rho^{n+1} dV = \sum \rho^n dV$$

$$\sum \rho^{n+1} gz dV = \sum \rho^* gz dV$$



Priestley algorithm

- Notes that loss of conservation arises from interpolation
- Compares low-order (specifically linear) interpolation with a high-order scheme (e.g. cubic or quintic)
- Argues that where these are different is where conservation will be lost
- Therefore adjusts high-order interpolated field proportionately to that difference
- Formally non-local but attempts to localize

Priestley (1993)



Monotonicity algorithm

- Higher-order interpolation scheme more accurate on smooth data
 - Cubic Lagrange is 3rd order accurate in space
- But applied to unsmooth data it will create overshoots and undershoots
- When this occurs high-order interpolation is not appropriate or sensible
- Could reduce the order progressively
- Pragmatic: limit the interpolated value to be bounded by the 8 values surrounding departure point

Bermejo and Staniforth (1992)



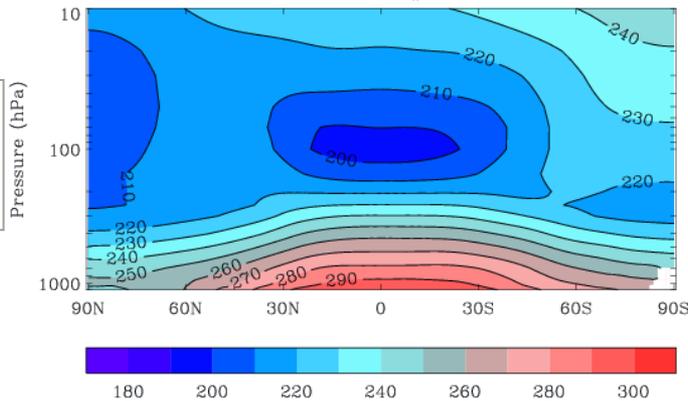
DOES IT MATTER WHAT WE DO?



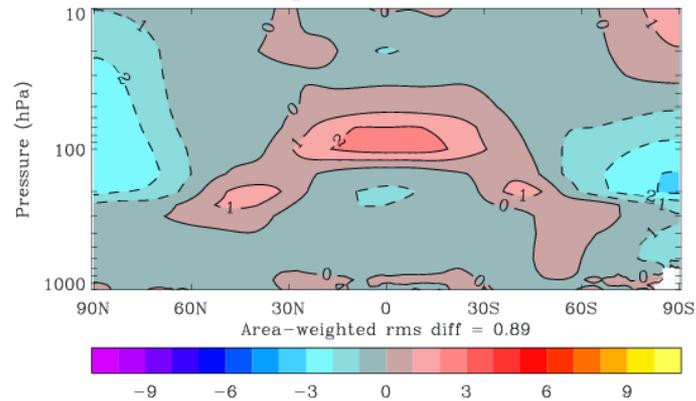
Temperature bias in 20 year AMIP run

ENDGame zonal mean temperature

a) Zonal mean Temperature for djf
ANHAH: GA5.0#95.11.2

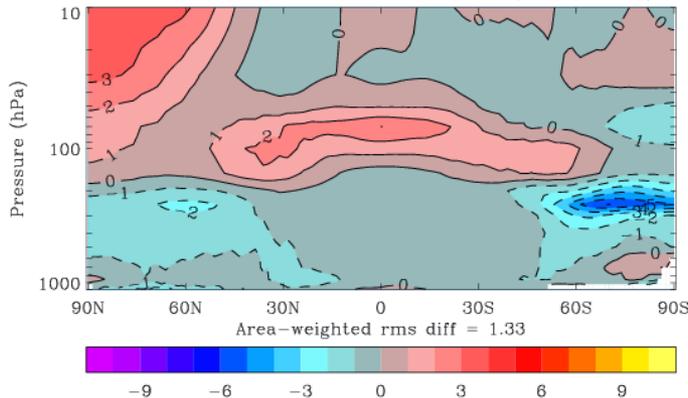


b) Zonal mean Temperature for djf
ANHAH: GA5.0#95.11.2 minus AMCHE: GA4.0



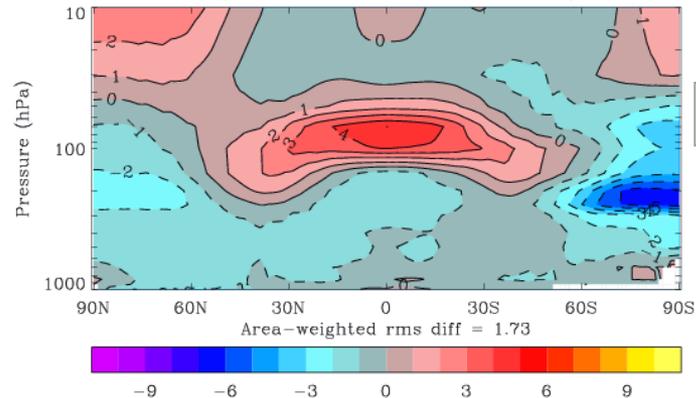
EG - ND

c) Zonal mean Temperature for djf
AMCHE: GA4.0 minus ERA-Interim (1989-2008)



ND - ERA

d) Zonal mean Temperature for djf
ANHAH: GA5.0#95.11.2 minus ERA-Interim (1989-2008)



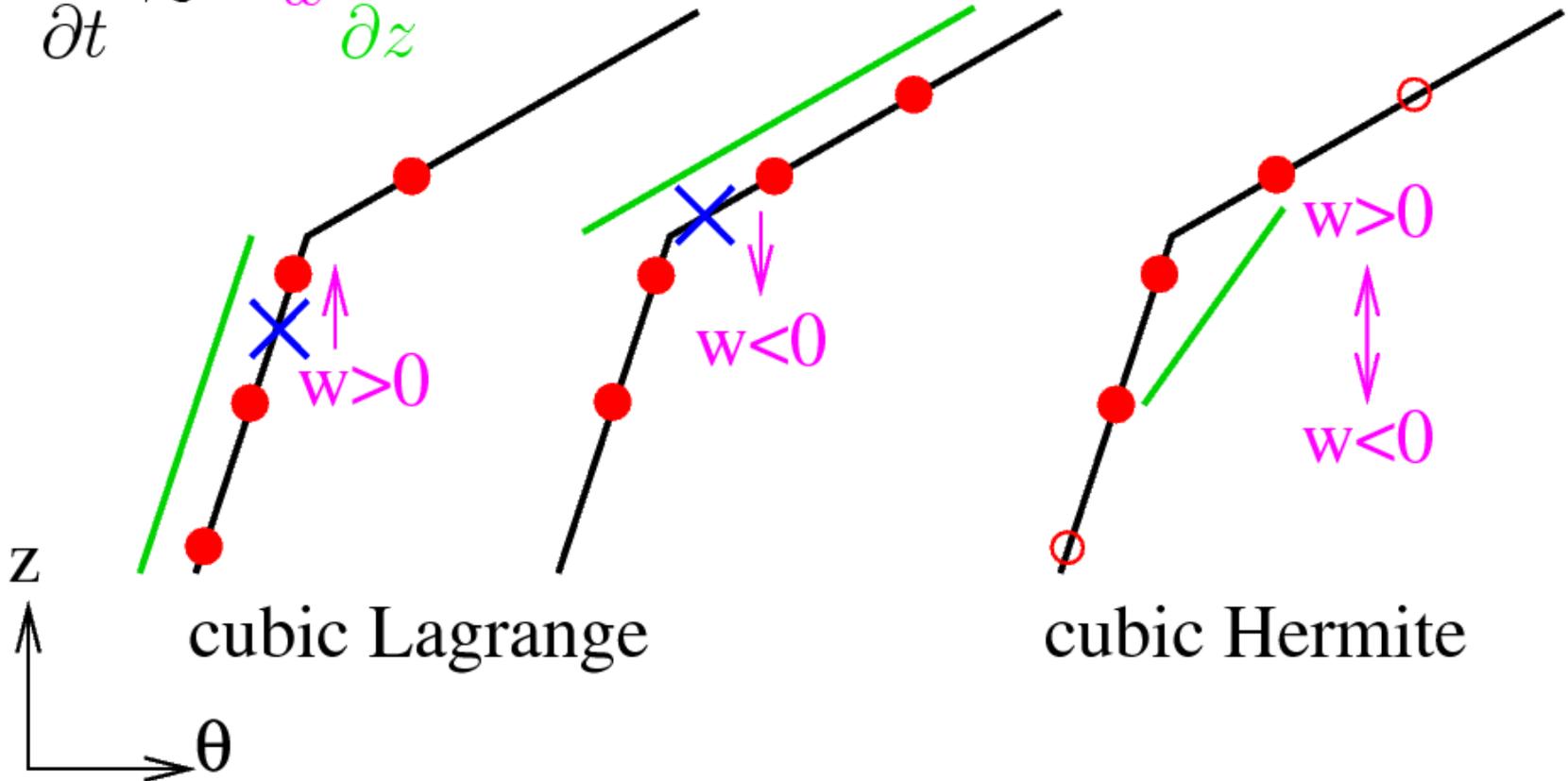
EG - ERA



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Why?

$$\frac{\partial \theta}{\partial t} \approx -w \frac{\partial \theta}{\partial z}$$



Chris Smith (Met Office)

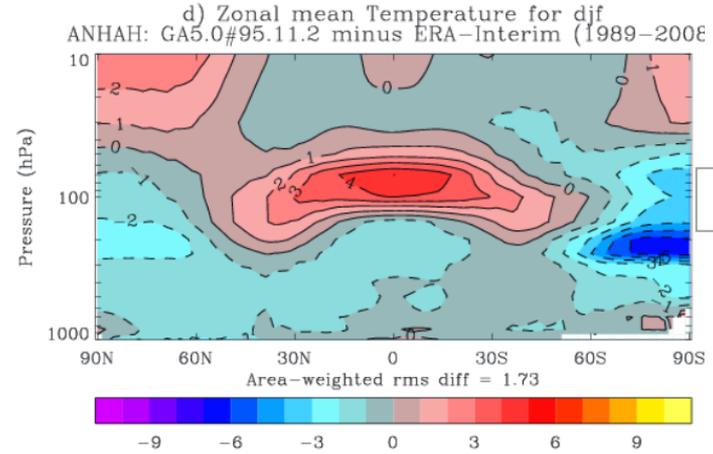
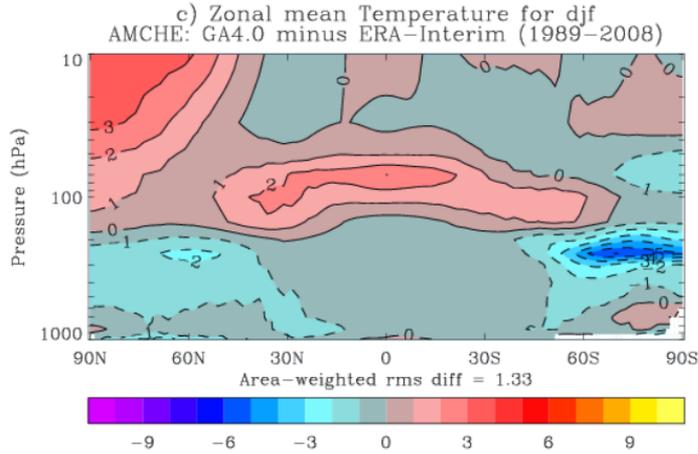
Impact of cubic Hermite + Priestley

Second-order centred

cubic Lagrange

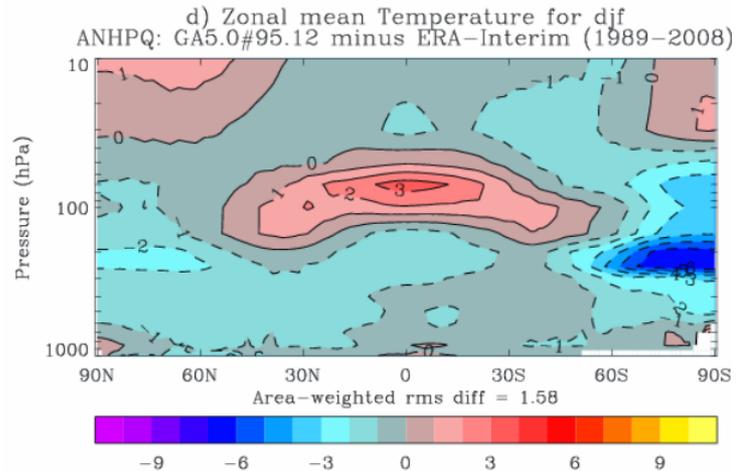
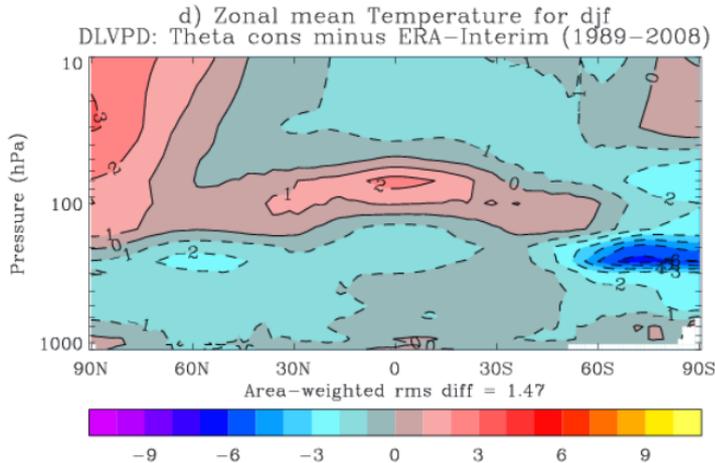
ND bias

EG bias



Priestley on potential temperature

cubic Hermite





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**SLICE:
SEMI-LAGRANGIAN INHERENTLY CONSERVATIVE
AND EFFICIENT**

RECOVERING CONSERVATION...

Conservative semi-Lagrangian

- Inherent conservation \Rightarrow must use density or concentration, ρ_X
- But instead of usual Eulerian flux form

$$\frac{\partial \rho_X}{\partial t} + \nabla \cdot (\mathbf{U} \rho_X) = 0$$

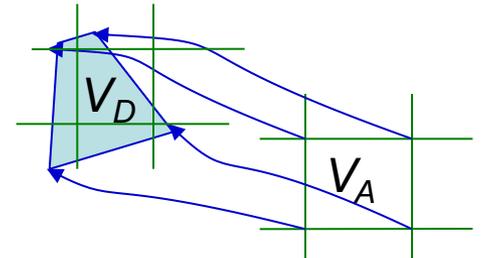
- Use Lagrangian form:

$$\frac{D}{Dt} \left(\int_V \rho_X dV \right) = 0$$

Conservative semi-Lagrangian

- Integrate along trajectory:

$$\int_{V_A} \rho_X^{n+1} dV = \int_{V_D} \rho_X^n dV$$



- Rearrange as:

$$\rho_X^{n+1} = \frac{1}{V_A} \left(\int_{V_D} \rho_X^n dV \right)$$



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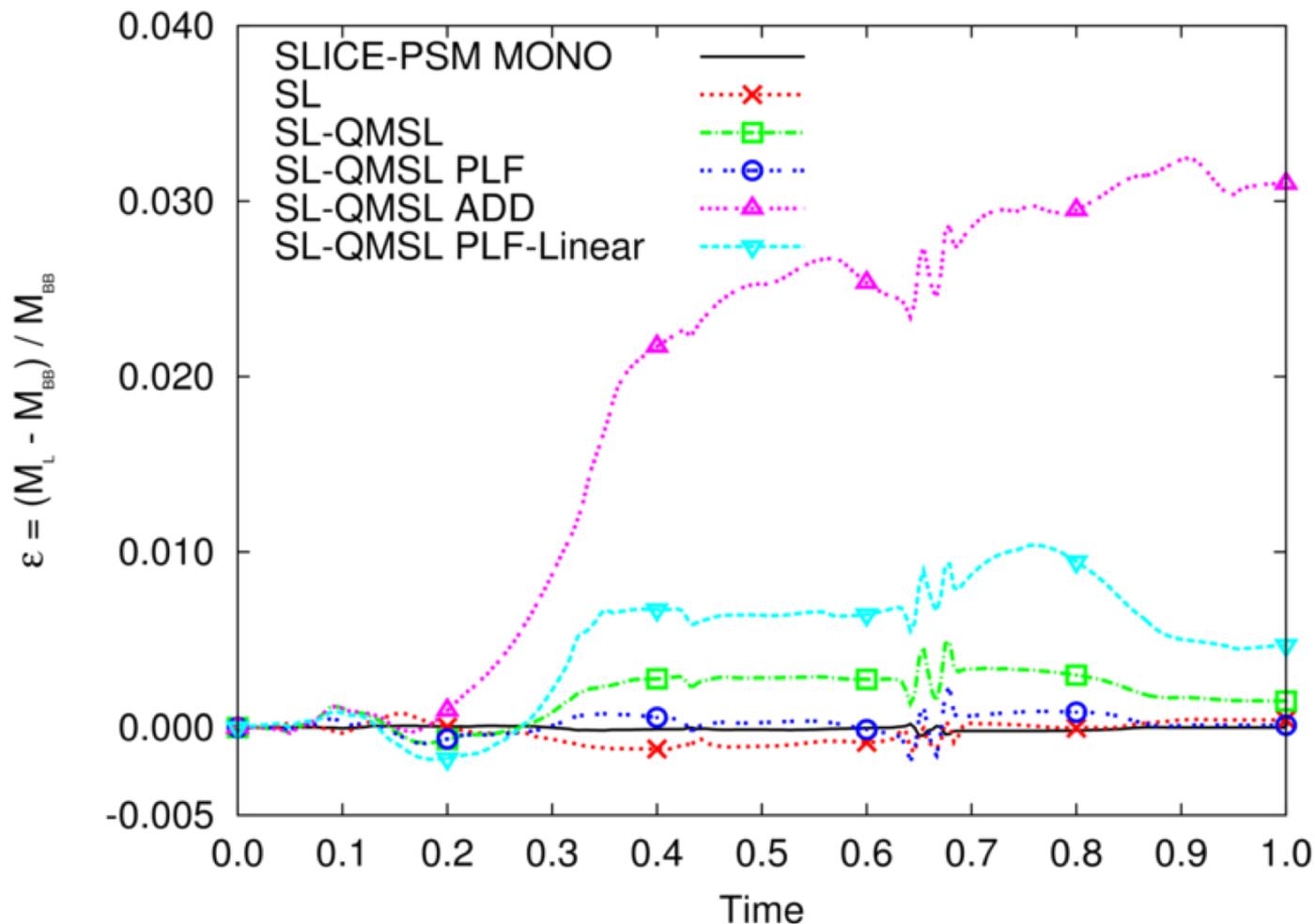
CONSERVATION IN LIMITED AREA MODELS...



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LAM Conservation (budget)

- SL alone good
- Monotonicity messes this up
- Conservation recovers accuracy
- And gives exact budget



PLF: Aranami, Davies and Wood (2014)

ZLF: Zerroukat & Shipway (2017)



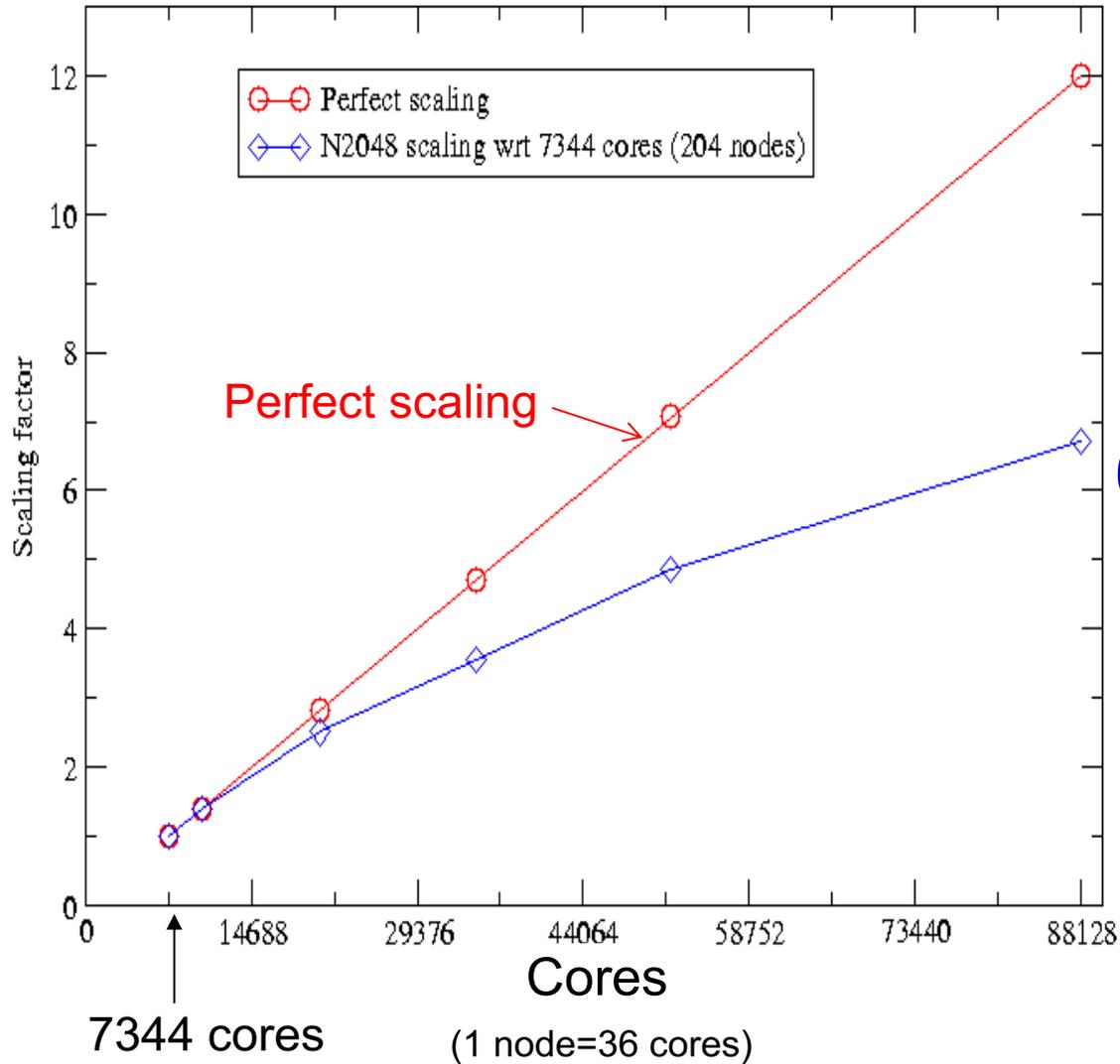
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GUNGHO INTO THE FUTURE!



Scalability

$$T_{7344}/T_N$$



ENDGame
6.5 km global

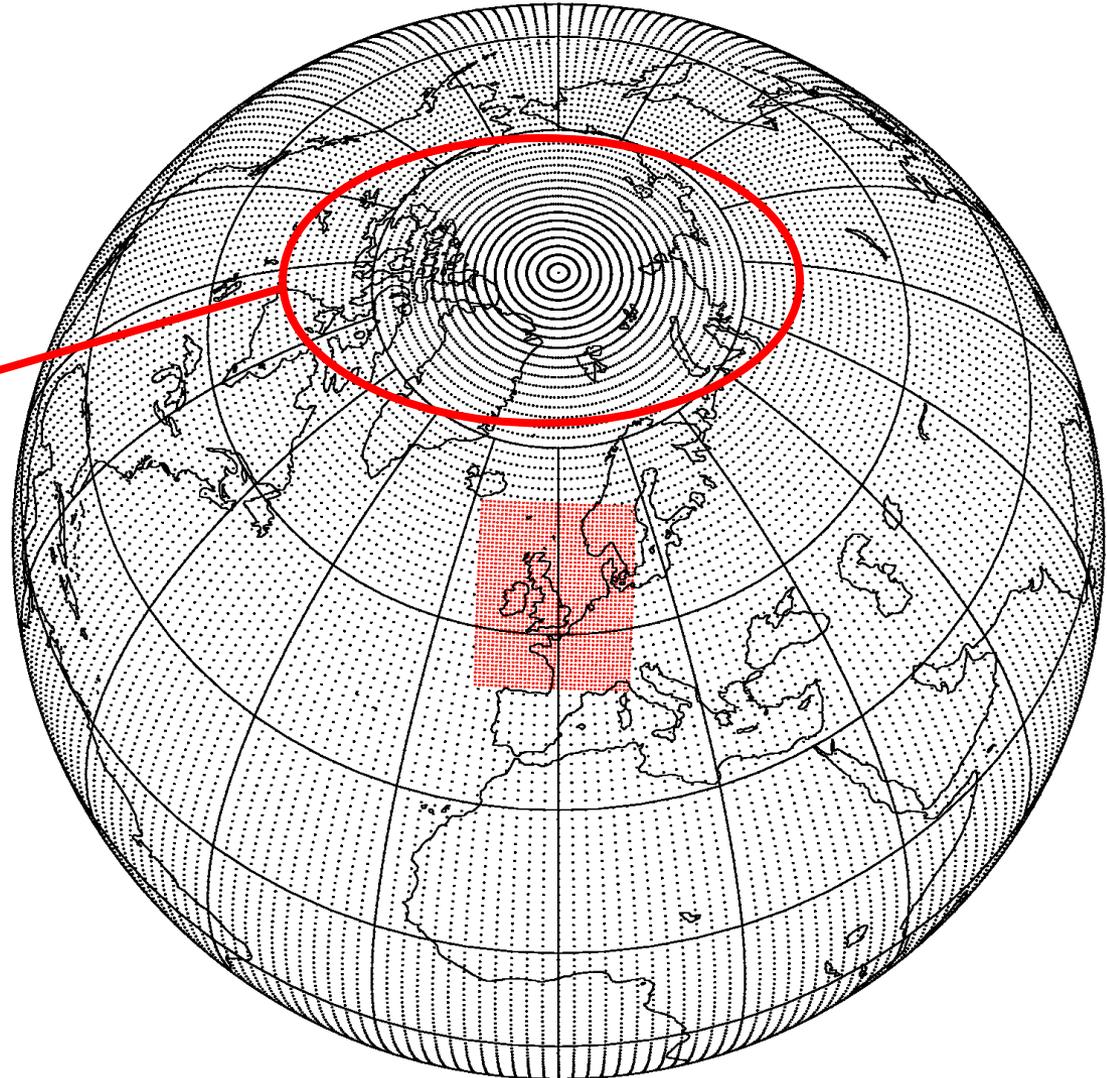
Paul Selwood & Andy Malcolm (Met Office)



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The finger of blame...

- At 17km resolution, grid spacing near poles = 35m
- At 10km spacing = 12m
- At 1km reduces to 12 cm!





GungHo!

Globally

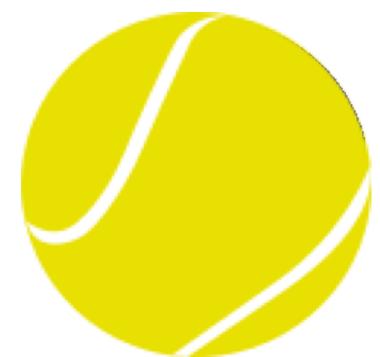
Uniform

Next

Generation

Highly

Optimized

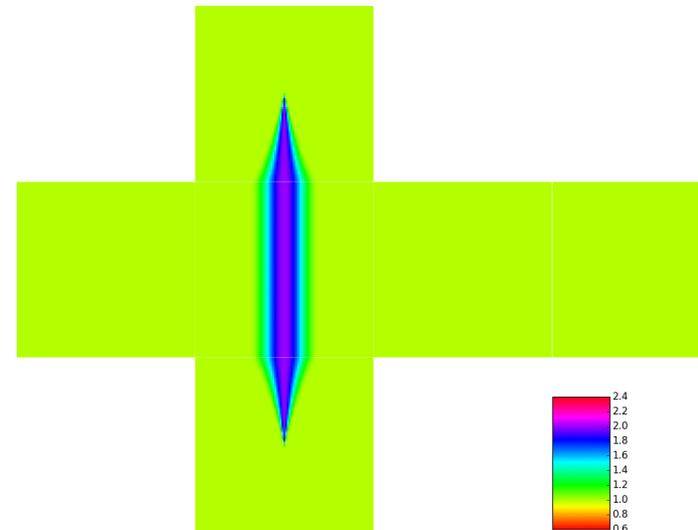


**Science & Technology
Facilities Council**

“Working together harmoniously”

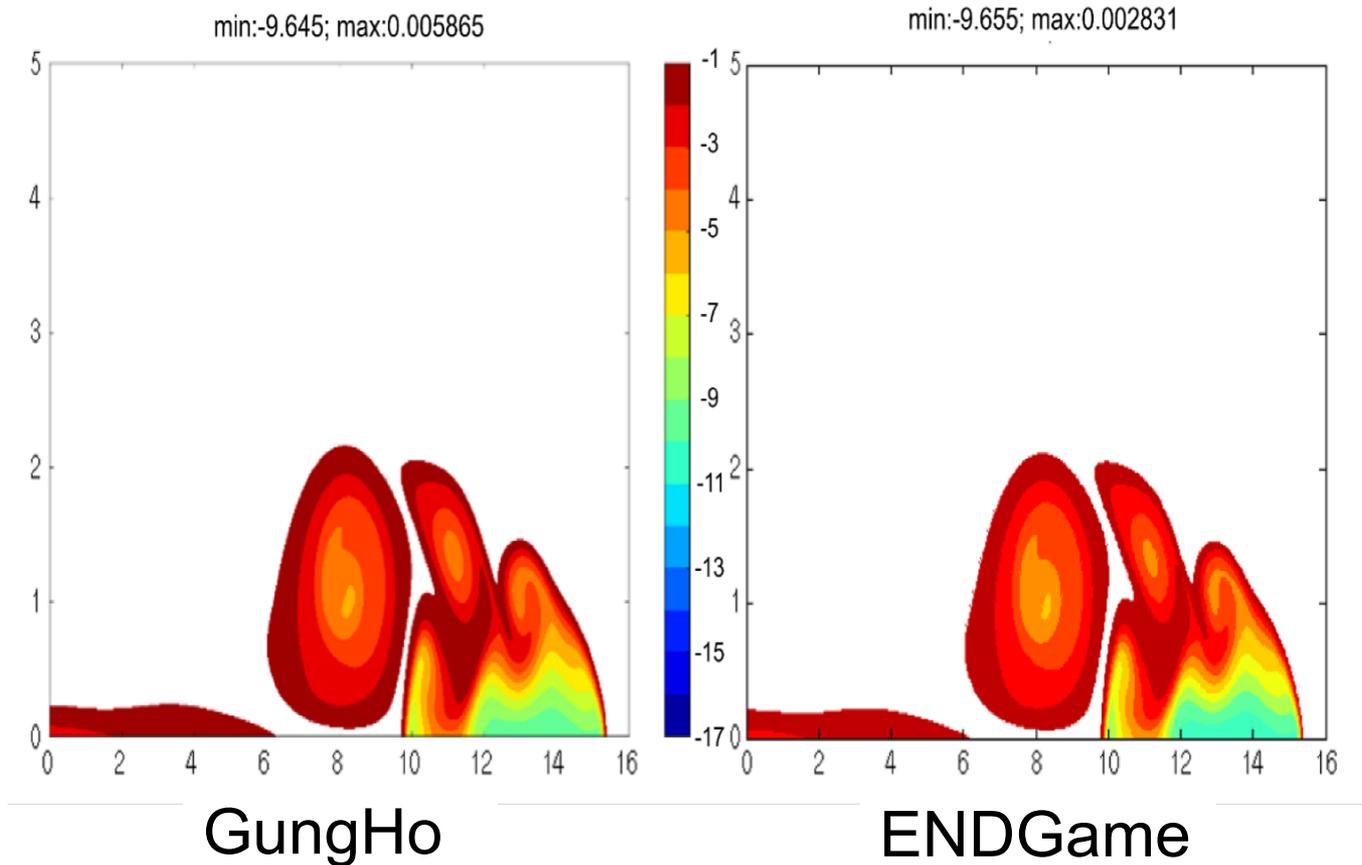
Where are we?

- Cubed-sphere is principal contender
- But grid non-orthogonal
- To maintain same accuracy using mixed finite-element spatial discretization...
- ...coupled with an ***Eulerian flux form*** transport scheme (either finite element or finite volume)
- Redesigning Unified Model
 - F2003
 - Separation of concerns - PSyKAI
- Targeting mid-2020's



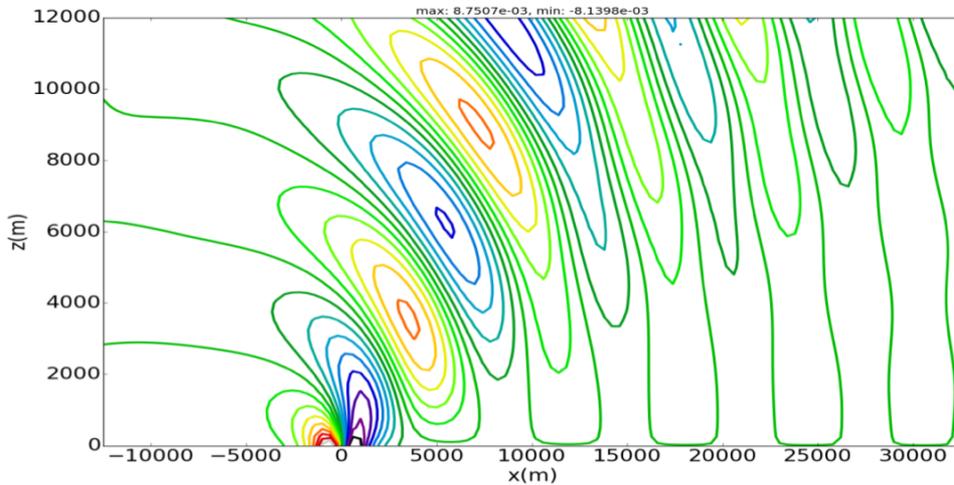
Straka cold bubble

Low Order Mixed FEM; GH = 50 m; EG = 50 m

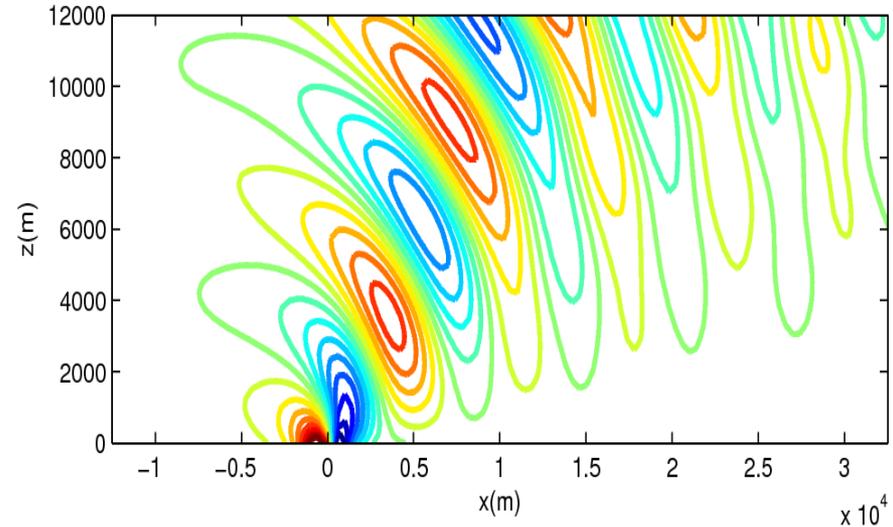


Orographic gravity waves

Low-order Mixed FEM; stratified flow over a small hill; uniform Cartesian domain



GungHo



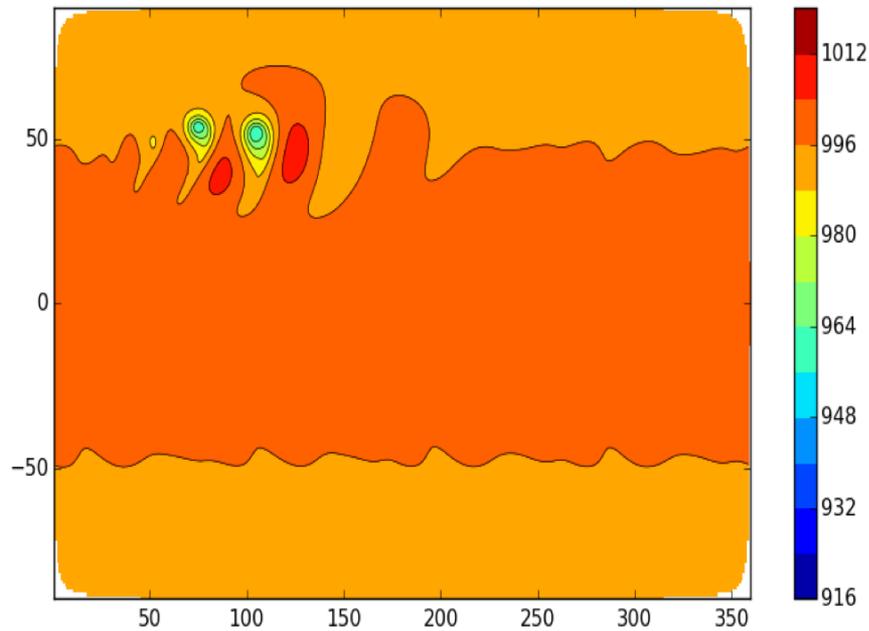
ENDGame



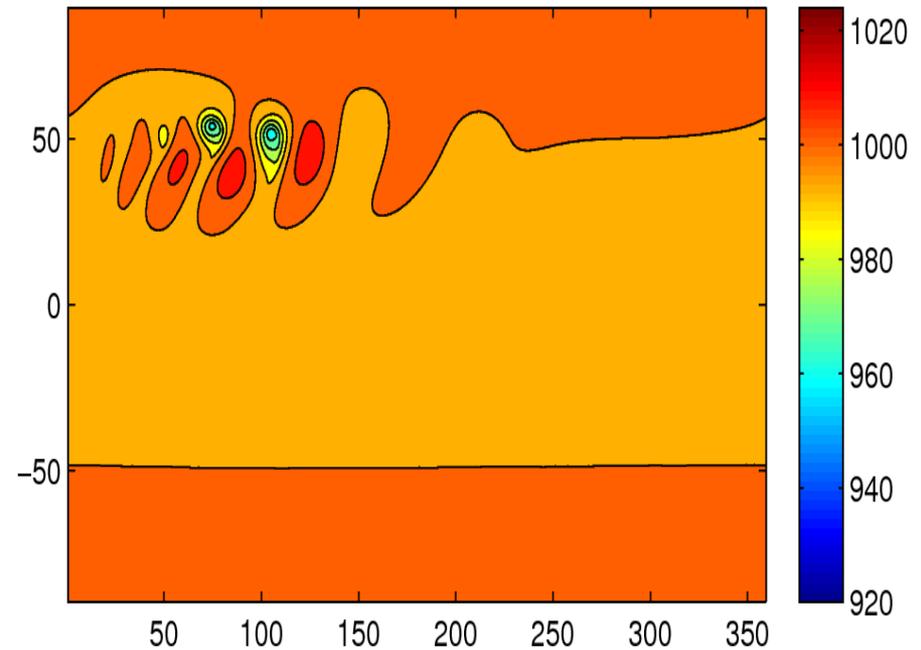
Baroclinic wave

Low Order Mixed FEM; GH = C96 ~ 1 degree; EG = 1 degree

Day 8 surface pressure



GungHo



ENDGame



Thank you!

Questions?

See extra slides for
Bibliography and **How**
to select options in UM

Bibliography

1. Aranami, K., Davies, T. & Wood, N. (2015) , A mass restoration scheme for limited area models with semi-lagrangian advection, *Q. J. R. Meteorol. Soc.* **141**, –. DOI:10.1002/qj.2482.
2. Bermejo, R. & Staniforth, A. (1992) , The conversion of semi-Lagrangian advection schemes to quasi-monotone schemes, *Mon. Wea. Rev.* **120**, 2622–2632.
3. Brown, A., Milton, S., Cullen, M., Golding, B., Mitchell, J. & Shelly, A. (2012) , Unified modeling and prediction of weather and climate: a 25-year journey, *Bull. Amer. Meteor. Soc.* **93**, 1865–1877.
4. Priestley, A. (1993) , A quasi-conservative version of the semi-Lagrangian advection scheme, *Mon. Wea. Rev.* **121**, 621–629.
5. Staniforth, A. & Côté, J. (1991) , Semi-Lagrangian integration schemes for atmospheric models - a review, *Mon. Wea. Rev.* **119**, 2206–2223.
6. Wood, N., Staniforth, A., White, A., Allen, T., Diamantakis, M., Gross, M., Melvin, T., Smith, C., Vosper, S., Zerroukat, M. & Thuburn, J. (2014) , An inherently mass-conserving semi-implicit semi-Lagrangian discretization of the deep-atmosphere global nonhydrostatic equations, *Q.J.R. Meteorol. Soc.* **140**, 1505–1520. DOI:10.1002/qj.2235.
7. Zerroukat, M., Wood, N. & Staniforth, A. (2002) , SLICE: A Semi-Lagrangian Inherently Conserving and Efficient scheme for transport problems, *Q. J. R. Meteorol. Soc.* **128**, 2801–2820.
8. Zerroukat, M. And Shipway, B. (2017) , ZLF (Zero Lateral Flux): A simple mass conservation method for semi-Lagrangian based limited area models, Submitted to *Q. J. R. Meteorol. Soc.*



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TRACER TRANSPORT OPTIONS IN ROSE

with thanks to Chris Smith



Interpolation options in Rose: $vn \geq 10.6$

Separate options for moisture and tracers ...

UM Advection ✕

monotone_scheme 1 1 0 0 1 < >
monotone scheme for (theta, moisture, winds, density and tracers)

high_order_scheme 8 7 1 1 7 < >
high order schemes for (theta, moisture, wind, density and tracers)

... with range of interpolation schemes

High order schemes:

- 0-Linear interpolation - no high order scheme
- 1-Cubic Lagrange interpolation
- 2-Quintic Lagrange interpolation
- 5-Bi-cubic Lagrange interpolation in the horizontal, linear interpolation in the vertical
- 7-cubic Lagrange interpolation in the horizontal, quintic in the vertical
- 8-LOCH: bi-cubic Lagrange in the horizontal; C1-Hermite cubic with quadratic derivative estimates in the vertical
- 9-HOCH: bi-cubic Lagrange in the horizontal; C1-Hermite cubic with quartic derivative estimates in the vertical

Close ✕



Interpolation options in Rose: $vn \geq 10.6$

... and new options for monotonicity

The screenshot shows the Rose software interface. The main window is titled 'Advection' and contains two configuration options:

- monotone_scheme**: monotone scheme for (theta, moisture, winds, density and tracers). The value is set to 1, 1, 0, 0, 1.
- high_order_scheme**: high order schemes for (theta, moisture, winds, density and tracers).

A help dialog box is open, titled 'Help for namelist:run_sl=monotone_scheme'. It lists the following monotone schemes:

- 0-No monotone scheme
- 1-Tri-linear Lagrange interpolation
- 3-Use PMF scheme
- 4-Use SPMF scheme (SPMF a stringent and more diffusive PMF)

The dialog box has a 'Close' button.



Conservation options in Rose: $vn \geq 10.1$

Tracer conservation now has the option to use the Priestley (1993) algorithm:

